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Cascade defense via navigation in scale free networks

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Abstract. In this paper, we investigate cascade defense and control in scale free networks via *navigation strategy*. It is found that with an appropriate parameter a, which is tunable in controlling the effect of degree in the navigation strategy, one can reduce the risk of cascade break down. By checking the distribution of *efficient betweenness centrality* (EBC) and the average EBC of vertices with degree k, the validity can be guaranteed. Despite the advantage of cascade defense, the degree based navigation strategy may also lead to lower network efficiency. To avoid this disadvantage, we propose a new navigation strategy. Importantly and interestingly, the new strategy can defend cascade break down effectively even without reducing the network efficiency. Distribution of the EBC and EBC-degree correlation of the new strategy are also investigated to explain the effectiveness in cascade defense.

PACS. 87.23.Ge Dynamics of social systems – 89.20.Hh World Wide Web, Internet – 89.40.Bb Land transportation – 89.75.Hc Networks and genealogical trees

1 Introduction

Complex networks can describe various systems in society, biology, transportation and communication etc. [1–4]. Since the seminal work on small world phenomena by Watts and Strogatz [5] and scale free property with degree distribution following the power law $P(k) \sim k^{-\lambda}$ by Barabási and Albert [6], a lot of effort has been devoted to understanding the structure and function of networks. In particular, features and dynamics of large scale transportation networks, such as the Internet [4], Power grids [5], World Wide Airports [7] and urban traffic systems [8,9], have recently attracted a large amount of interest from the physics community due to their importance in our daily life. For a transportation network, how to defend against intentional attacks and obtain more robustness, how to achieve higher efficiency in handling and navigating agents, and how to control traffic congestion, etc, are all important problems. There is a great need to understand the traffic dynamics on complex networks.

In previous studies, various navigation strategies on networks have been investigated. Among them, two navigation strategies are particularly useful and efficient. One is based on traditional shortest path, and the other is based on the degree of vertices. By using the shortest path based navigation strategy, many studies of traffic dynamics have been under taken, such as load distribution [10], cascading failure [11–13], overload phenomenon [14,15], microscopic time fluctuations [16,17], traffic congestions [18,19], optimal distance [20], and optimal topology which may avoid congestion [21]. In all of these studies, the route for each agent to choose is according to the shortest path it may take. Recently, progress was achieved using the degree based navigation strategy [22–27]. In that strategy, agents are navigated according to the degree of the vertices they may choose in the next time step.

The degree based navigation strategy can be briefly reviewed as follows for consistency and completeness of this paper. The strategy is introduced because the path with shortest length is not necessarily the quickest way in many transportation and communication systems, especially on heterogeneous networks. In scale free networks, vertices with large degree are more likely to suffer traffic congestion, thus an agent may spend more waiting time to pass through on average. By passing those hub nodes with high degree and choosing other less congested routes, the agents may reach their destination quicker than taking the shortest path. Based on the phenomenon, the *efficient path* is proposed by Yan et al. [23]. For any path between vertex i and j as $P(i \to j) := i \equiv x_0, x_1, ...x_{n-1}, x_n \equiv j$, denote

$$L(P(i \to j) : a) = \sum_{i=0}^{n-1} k(x_i)^a, \qquad (1)$$

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the efficient path between i and j is the route that makes the sum $L(P(i \to j) : a)$ minimum. Obviously, $L_{\min}(a = 0)$ recovers the traditional shortest path length. So the efficient betweenness centrality (EBC) can be defined as

$$g^a(v) = \sum_{s \neq t, s \neq v} \frac{\sigma_{st}^a(v)}{\sigma_{st}^a} \quad (2)$$

where σ_{st}^a is the number of efficient paths for a given a going from s to t and $\sigma_{st}^a(v)$ is the number of efficient paths for a given a going from s to t and passing through v. In the following studies [22,24–26], it was shown that on scale free networks, the degree based navigation strategy performs better than the shortest path. And it is also found that the optimal value to gain the most efficient routing strategy is $a = 1$.

Because most real transportation networks, such as the Internet and urban traffic systems, usually display scale free degree distribution, a degree based navigation strategy may perform better than a shortest path based one on these networks. With their feature of robustness and fragility, scale free networks can resist random attacks effectively but break down due to serious intentional attacks. Even if a single node is removed, major changes might occur due to its heterogeneous load distribution. Specifically, if the load (or betweenness centrality) of that removed node is relatively large, the initial change is likely to affect the network significantly due to overload failures of other nodes. The effect is called cascade break down and was firstly investigated by Motter and Lai [11]. Since then, many groups have studied the dynamics of cascade failures on networks [12,13,19,28–31]. However, their investigations are based on the definition of load, or betweenness centrality. That means the agents on the network are navigated via the shortest path based strategy. As discussed above, the degree based navigation strategy is useful and performs well especially on heterogeneous networks. If the agents select their routes via a degree based navigation strategy, will the cascade break down still take place due to deliberate attacks? And if the cascading still happens, is there any approach to defend against it? These problems are interesting in understanding the role of navigation strategy in cascading dynamics but still unclear. In this paper, we try to fill this gap by investigating the cascade break down and defense via degree based navigation.

2 Cascade via degree based navigation strategy

The damage caused by cascading is usually quantified by the relative size of the largest connected component G defined as [11],

$$G = N'/N \quad (3)$$

where N and N' are the number of nodes in the largest component before and after the cascade failure. To measure the efficient behavior of the network, Latora et al. [32] introduced a definition called network efficiency. The efficiency e_{ij} between nodes i and j is inversely proportional to the shortest distance d_{ij}, $e_{ij} = 1/d_{ij}, \forall i, j$. If there is no path between the nodes i and j, $d_{ij} = +\infty$ and $e_{ij} = 0$. The average efficiency of the network can be defined as,

$$E = \frac{1}{N(N-1)} \sum_{i \neq j} e_{ij} \quad (4)$$

where N is the size of the network.

Here we define the load $D_i(t)$ on node i at time t as the total number of paths passing through that node via a specified navigation strategy. Each node is assigned a finite capacity as given in the ML model [11],

$$C_i = (1 + \alpha) D_i(0), \ i = 1, ..., N, \quad (5)$$

where $\alpha \geq 0$ is the tolerance parameter. Next, we will investigate the cascade break down triggered by the removal of a single node, randomly or intentionally. The main differences with respect to previous models are as follows: (1) the navigation strategy we adopt here is the degree based one. That is totally different to traditional ones; (2) one of our main focuses is on the effectiveness of defending against cascade break down via the degree based navigation strategy; (3) the damage caused by the cascade break down is also quantified in terms of the network efficiency E.

The network adopted here is the classical BA model network with mean degree $\langle k \rangle = 3$, scaling exponent $\lambda = 3$ and network size $N = 1000$ [1,6]. Firstly, we investigate cascading failure in scale free networks as triggered by intentional removal with largest degree. As shown in Figure 1a, cascade does not take place when $a > 0$ while it is aggravated as $a < 0$. By checking the EBC in $a > 0$ cases, we find the EBC at the nodes with largest degree is $N - 1$. Actually, in case $a = 1$, about 3 per-thousand and in case $a = 2$, about 2.5% nodes with largest degree are bypassed by all agents on the network except those agents whose destinations are the hub nodes. In fact, the nodes with largest degree, or hub nodes, are usually important nodes and perform key functions in the system. It is obviously unadvisable to bypass these nodes for all agents. This phenomenon also leads low network efficiency directly as confirmed as follows in Figure 1b. To avoid the lack of network efficiency, in the next section, we will introduce a new navigation strategy to avoid the disadvantages. By taking the strategy with parameter $a < 0$, the agents prefer hub nodes much more than a traditional shortest path based navigation strategy, which may lead to heavier congestion. So the system may be under more risk of cascade break down. As is reported in Figure 1a, in the $a < 0$ cases, even though the tolerance parameter reaches $\alpha = 1$, the system may still break down. Furthermore, the network efficiencies after the cascading process are measured and the results are presented in Figure 1b. As is expected, the efficiency of the network achieves its maximum at $a = 0$, that is the case that all agents are navigated by the traditional shortest path based method. For the degree based navigation strategy, no matter whether $a > 0$ or not, network efficiency is reduced. It is also found that in the $a = 1$ case, the network efficiency reaches it

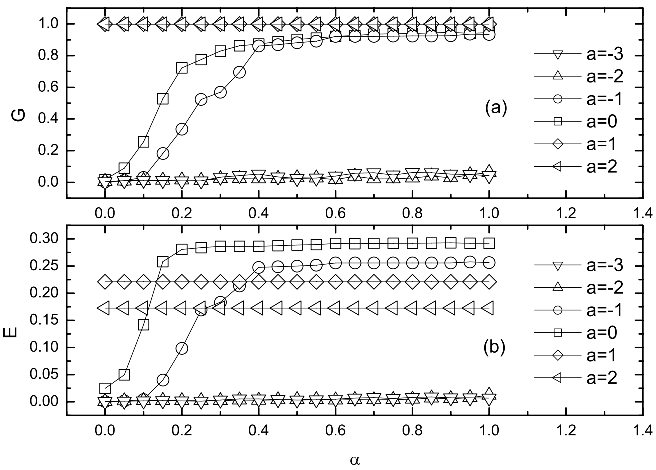

Fig. 1. Cascading failures via degree based navigation strategy as triggered by intentional removal with the largest degree, (a) is the relative size of the largest connected component G as a function of tolerance parameter α; (b) is the network efficiency E as a function of the parameter α. The networks are generated with size $N = 1000$ and the average degree is $\langle k \rangle = 3$.

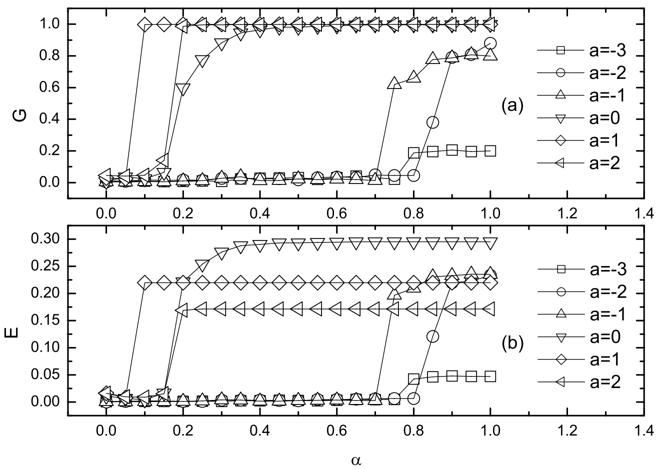

Fig. 2. Cascading failures via degree based navigation strategy as triggered by intentional removal with the largest load, (a) is the relative size of the largest connected component G as a function of tolerance parameter α and (b) is the network efficiency E as a function of the parameter α. The networks are generated with size $N = 1000$ and the scaling exponent is $\lambda = 3$.

maximal value without the risk of cascade break down. So we can deduce that $a = 1$ is the optimal value in cascade defense while maintaining the network efficiency at a high level.

We next investigate cascade break down caused by removal of a single node chosen among those with highest load. The results are shown in Figure 2. In Figure 2a, the relative size of the largest connected component G as a function of α is presented. We can see that in $a > 0$ cases, the risk of cascade break down is reduced considerably. As is illustrated, in the case $a = 1$, the critical value of α_c reaches its minimum. And in the $a < 0$ cases, the cascading failure is aggravated as reported in the figure. In the $a < 0$ cases, even though the tolerance parameter reaches $\alpha = 0.8$, the system may still be at risk of break down. From Figure 2b, we can also find that the network efficiency is reduced due to the degree based navigation strategy and $a = 1$ is still the optimal value in defending against cascade break down while maintaining the network efficiency at a relatively high level. The value is the same as in the case of removal with largest degree.

Cascade break down caused by random removal is also considered and the results are shown in Figure 3. Unlike the removal with largest degree or load, with the parameter $a < 0$, the cascade break down may not occur if a node is removed randomly. By checking the EBC in $a < 0$ cases, we find that the EBC of a large amount of nodes with low degree are $N - 1$. In fact, it is checked that 65% ($a = -1$), 75% ($a = -2$) and 80% ($a = -3$) nodes of the system are bypassed by all agents, except the instance of being destination of an agent. What is more, as confirmed as follows, the strategies with $a < 0$ strengthen the heterogeneousness of load. By taking such strategies, most of agents are navigated through the hub nodes. On the contrary, the navigation strategy with parameter $a > 0$ may lead to a more homogeneous load distribution. As

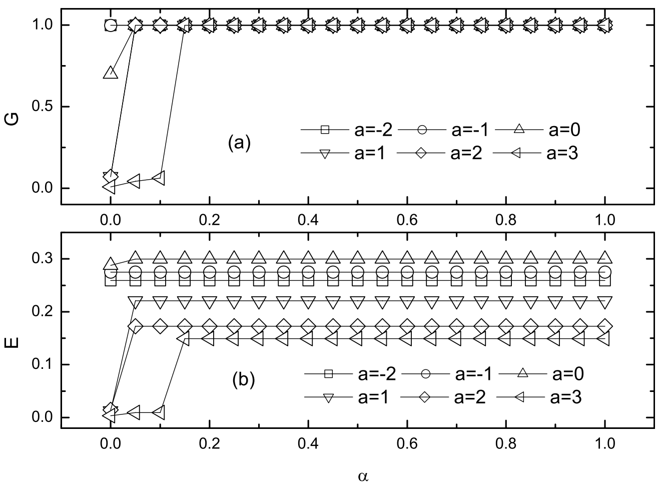

Fig. 3. Cascading failures via degree based navigation strategy as triggered by random removal, (a) is the relative size of the largest connected component G as a function of tolerance parameter α and (b) is the network efficiency E as a function of the parameter α. The networks are generated with size $N = 1000$ and the scaling exponent is $\lambda = 3$.

is illustrated in Figure 3a, when a node is removed randomly, major changes in the balance of loads may occur. And from Figure 3b, we can also find that the network efficiency is reduced because of the degree based navigation strategy.

Since it is proposed that heterogeneous load distribution is one of main reasons of cascading failure [11], we investigated the statistical properties of EBC. The EBC distribution are shown in Figure 4. From the figure, we can see that the distribution of traditional betweenness centrality follows a power law, which confirms the results

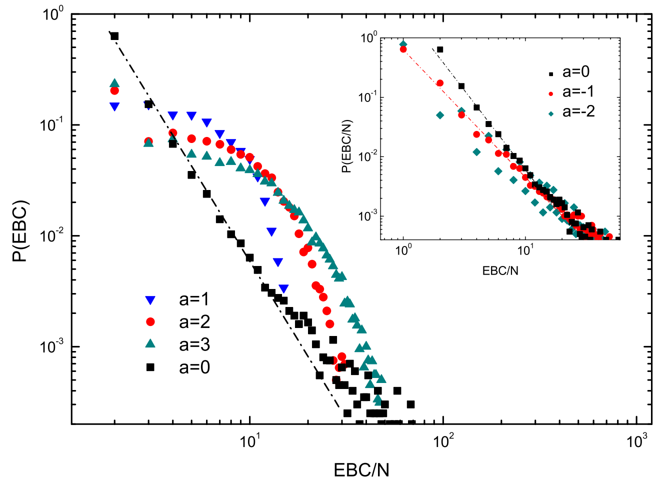

Fig. 4. Distribution of efficient betweenness centrality with various parameter $a > 0$ of scale free networks. In the inset, the efficient betweenness centrality distribution with parameter $a < 0$ are presented. Each curve corresponds to the average over 20 realizations of network with size $N = 1000$ and scaling exponent $\lambda = 3$. The data are logarithmically binned.

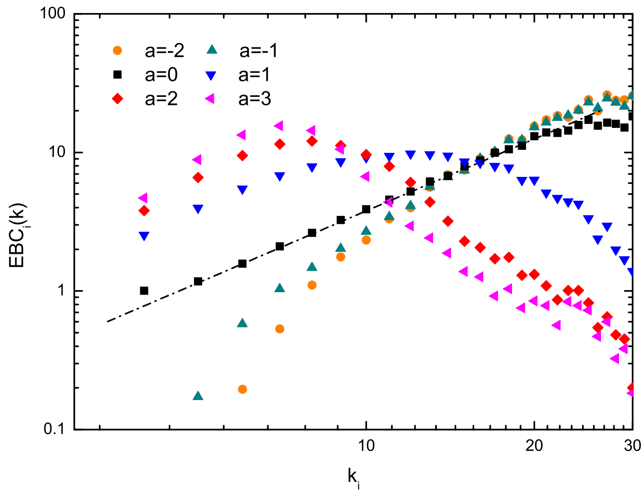

Fig. 5. Average efficient betweenness of vertices with degree k for various parameter a in a log-log scale. The results are averaged over 20 realizations of network with size $N = 1000$.

in Goh et al. [10,33]. When $a > 0$, the EBC has a narrower distribution. That means the heterogeneousness of the distribution is weakened. However, in $a < 0$ cases, the EBC distribute is broader than that of the $a = 0$ case. That corresponds to a strengthened heterogeneousness. Since the heterogeneousness of load is one of the main reasons for the cascade break down, we consider that the different cases of EBC distribution can explain the role of navigation strategy in cascading.

We next examine the average EBC of the vertices with degree k to understand the effectiveness of the cascade defense via degree based navigation strategy. The results are shown in Figure 4. As is illustrated, in the $a = 0$ case, the correlation between EBC (traditional betweenness centrality) and degree follows a power law, which confirms the results given in [10]. In $a < 0$ cases, the EBC-degree correlations increase even faster, while in $a > 0$ cases, the EBC-degree correlations decay at large k. That means those navigation strategies with parameter $a > 0$ may lead to low load or EBC at hub nodes. Meanwhile, in $a < 0$ cases, the fast growing EBC-degree correlations correspond to larger EBC at hub nodes. Those hub nodes are then likely to bear more traffic congestions. That can explain why the degree based navigation strategy may act against cascade break down when $a > 0$ and aggregate cascade failure when $a < 0$.

3 New navigation strategy

As mentioned above, the degree based navigation strategy with appropriate parameter a may defend against cascade break down effectively. On the other hand, the network efficiency E may be reduced by such strategies. When congestion does not occur, traffic flows on the whole network are in free state. In this case, bypassing the hub nodes may lengthen the path length between two nodes in the network meaninglessly. What is more, in $a > 0$ cases, the EBC of some hub nodes are $N - 1$, while in $a < 0$ cases, even more than 50% nodes with low degree are bypassed by all agents. Since the hub nodes usually have more powerful handling abilities, bypassing these nodes by all agents of the system is obviously inappropriate. Then how to achieve higher network efficiency with robustness against intentional attack is in need. In this section, we propose a new navigation strategy which combines the shortest path information and the degree information together. For any path between vertex i and j as $P(i \to j) := i \equiv x_0, x_1, ...x_{n-1}, x_n \equiv j$, denoted

$$L(P(i \to j) : q) = \sum_{i=0}^{n-1} (1 - q + qk(x_i)/k_{\max}), \qquad (6)$$

the efficient path between i and j here we introduced is the route that makes the sum $L(P(i \to j) : a)$ minimum, where $0 \le q \le 1$ is a parameter and $k_{\max}$ is the largest degree of the network. Obviously, when $q = 0$, the new strategy is the traditional shortest path based one and when $q = 1$, the strategy is just the degree based navigation strategy with parameter $a = 1$. The corresponding efficient betweenness centrality can be defined as follows,

$$g^q(v) = \sum_{s \ne t, s \ne v} \frac{\sigma_{st}^q(v)}{\sigma_{st}^q} \qquad (7)$$

where σ_{st}^q is the number of efficient paths for a given q going from s to t and $\sigma_{st}^q(v)$ is the number of efficient paths for a given q going from s to t and passing through v. Importantly, by using this navigation strategy, it can defend against cascade failures effectively even without reducing network efficiency.

Cascading dynamics are simulated extensively to understand the effect of the new strategy. The networks we

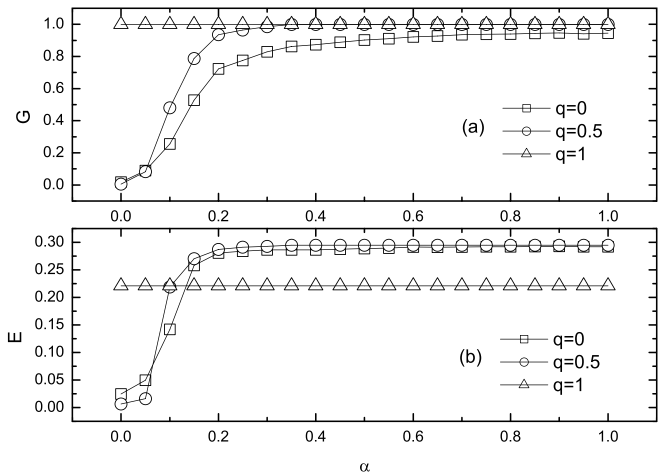

Fig. 6. Cascading failures via the new navigation strategy as triggered by removal with the largest degree, (a) is the relative size of the largest connected component G as a function of tolerance parameter α and (b) is the network efficiency E as a function of the parameter α. The networks are generated with size $N = 1000$ and the scaling exponent is $\lambda = 3$.

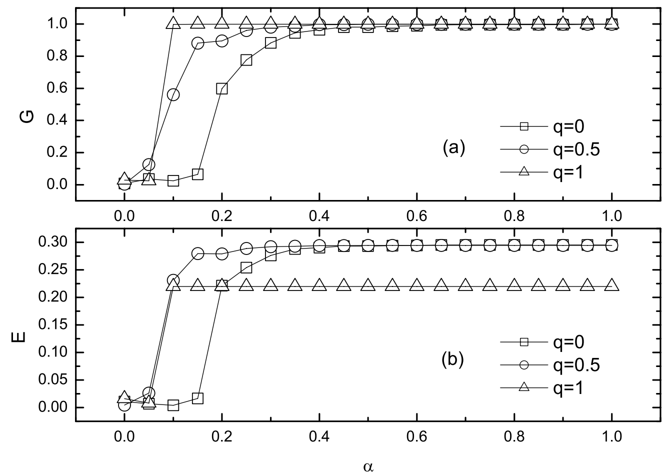

Fig. 7. Cascading failures via the new navigation strategy as triggered by removal with the largest load, (a) is the relative size of the largest connected component G as a function of tolerance parameter α and (b) is the network efficiency E as a function of the parameter α. The networks are generated with size $N = 1000$ and the scaling exponent is $\lambda = 3$.

adopt are also the BA model networks with the same size in the previous section. Figure 6 shows the results of cascade break down caused by intentional removal with largest degree. As reported in Figure 6a, the new navigation strategy indeed reduces the risk of cascade failure. Together with Figure 6b, we can see that the navigation strategy does not reduce the network efficiency. Figure 7 reports the cascading failure caused by removal with largest load. As is illustrated in Figure 7a, the robustness of the network are enhanced considerably. It is found that by using the new navigation strategy, the system can defend against cascade break down with a much smaller tolerance parameter α. And as shown in Figure 7b, the network efficiency does not need to be reduced either. Because the degree based navigation strategy with $a > 0$ does not perform so well in cascading caused by random removal, does the new strategy defend against random attacks effectively? In Figure 8, the results of cascade break down caused by random removal are given. As shown in Figure 8a, we can see that the system may resist cascading with a relatively small tolerance parameter α, say less than 0.05, which is acceptable. In addition, the network efficiency does not need to be reduced as illustrated in Figure 7b. Furthermore, it is checked that all the new EBC corresponding to the new navigation strategy are larger than $N - 1$, which means no nodes are bypassed by all agents. That may be more of an advantage in navigating agents in real transportation systems than in pure degree based ones.

To have a deeper insight into the relationship between shortest path based navigation strategy and the degree based navigation strategy, we investigated the network efficiency as a function of the parameter q. The results are presented in Figure 9. When q is relatively small, say $q \leq 0.5$, the network efficiency does not reduce apparently.

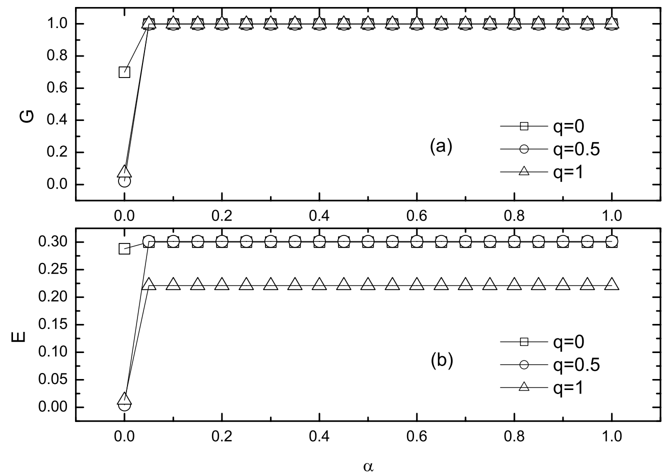

Fig. 8. Cascading failures via the new navigation strategy as triggered by random removal, (a) is the relative size of the largest connected component G as a function of tolerance parameter α and (b) is the network efficiency E as a function of the parameter α. The networks are generated with size $N = 1000$ and the scaling exponent is $\lambda = 3$.

However, when q increases beyond the critical value 0.5, the network efficiency reduces quickly. We consider that the phase transition is because of the heterogeneous degree distribution of scale free networks. In fact, from the combination (6), the hub nodes give more impact on the new EBC while those nodes with low degree contribute little to the combination. Then if q is relatively small, agents choose their route almost according to the shortest path and only those hub nodes are likely to be bypassed. That does not affect the network efficiency apparently. However, with q increased large enough, the nodes with low degree have non-negligible impact on the new navigation strategy. Then the network efficiency decreases quickly as illus-

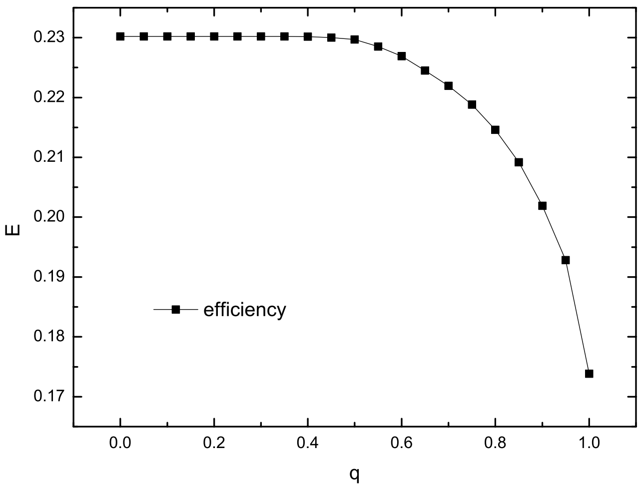

Fig. 9. Phase transition of the network efficiency E as a function of the parameter q in the new navigation strategy. One can easily check that the critical value is $q = 0.5$.

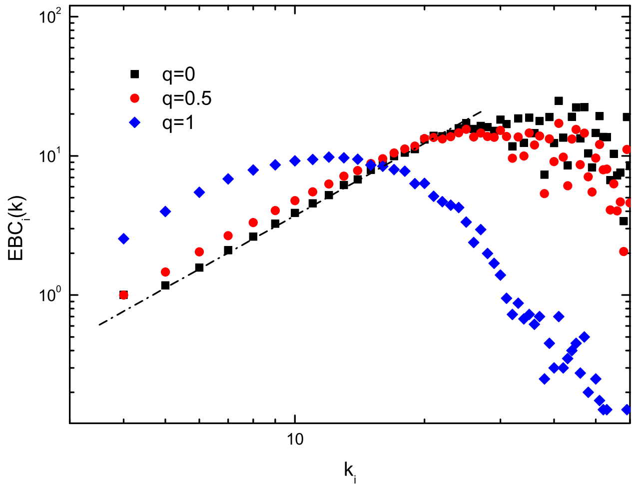

Fig. 11. The BC-degree correlation of the new navigation strategy. The results are averaged over 20 realizations with networks generated with size $N = 1000$ and scaling exponent $\lambda = 3$.

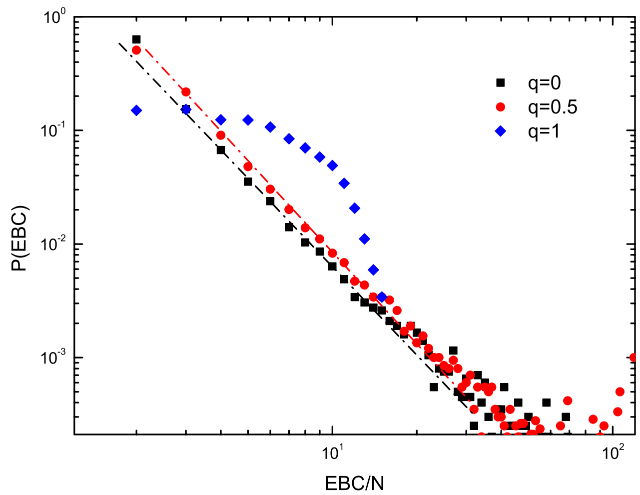

Fig. 10. Distribution of betweenness centrality when agents are navigated via the new strategy. The results are averaged over 20 realizations with networks generated with size $N = 1000$ and scaling exponent $\lambda = 3$.

trated. From the features described above, we can achieve the most effective strategy to defend against cascade break down with holding a high network efficiency at the critical value $q = 0.5$.

The new EBC distribution and new EBC-degree correlation are also investigated. Figures 10 and 11 give the results. From Figure 10, we can check that the distribution of new EBC at $q = 0.5$ is narrower than the traditional betweenness centrality, which expresses a less heterogeneous load distribution. From Figure 11, one can check that at $q = 0.5$, the EBC of nodes with large degree is lower than the traditional one. The features can also explain the effectiveness of cascade defense.

4 Conclusions

In conclusion, we investigate the cascading defense via navigation strategy and the simulation results show that the degree based navigation strategy with parameter $a > 0$ can reduce the risk of cascade break down caused by intentional attacks effectively. In cascading caused by largest degree or load, the optimal value of the parameter in defense, while maintaining a relatively high network efficiency, is $a = 1$. The distribution of EBC and average EBC of vertices with degree k are investigated to explain the reduced risk of cascade break down via degree based navigation. However, despite the advantage in defending against cascade break down, degree based navigation strategy may also reduce the network efficiency. To defend against cascade break down while maintaining a high network efficiency, a new navigation strategy is proposed which combines the traditional shortest path information and the degree of the vertices. Simulation results show that the new navigation strategy performs well in cascade defense without damaging the network efficiency. Furthermore, a critical value of the parameter in network efficiency transition is obtained. The EBC distribution and EBC-degree correlation of the new navigation strategy can also successfully explain this phenomenon.

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